

# The Nordtvedt Effect in Rotational Motion

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## Abstract

Rotational motion of extended celestial bodies is discussed in the framework of the Parametrized Post-Newtonian (PPN) formalism with the two parameters  $\gamma$  and  $\beta$ . A local PPN reference system of a massive extended body being a member of a system of  $N$  massive extended bodies is constructed. In the local PPN reference system the external gravitational field manifests itself only in the form of tidal potentials. Rotational equations of motion, which are then derived in the local reference system, reveal a special term in the torque analogous to the Nordtvedt effect in the translational equations of motion: it is proportional to  $4\beta - \gamma - 3$ , to the acceleration of the body relative to the global PPN reference system and to some quantity characterizing the distribution of inertial gravitational energy within the body. This term is a direct consequence of the violation of the Strong Equivalence Principle in alternative theories of gravitation.

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## I. INTRODUCTION

In present paper we deal with rotational motion of a celestial body being a member of a system of  $N$  extended bodies in the PPN formalism. It is well known that rotational motion of non-isolated extended celestial bodies is a subtle point. In General Relativity this problem has been treated in the post-Newtonian approximation by Fock [1] who used one global reference system to describe both translational motion of  $N$  extended rotating bodies and their rotational motion. The resulting equations of motion derived by Fock showed complicated couplings between translational motion of the bodies and their rotation. However, most of these couplings were only spurious coordinate effects which were later shown to vanish when using physically adequate local reference systems to describe the local dynamics (e.g., rotation) of each body of the system (see [2] for further comments and references). The idea of the local reference systems is to introduce a reference system for a body in which the influence of external masses is effaced as much as possible. In General Relativity the local reference systems have been discussed in great details by Brumberg and Kopeikin [3,4] (see also [5]) and Damour, Soffel and Xu [6,7,2]. In particular it has been shown that the local reference systems in General Relativity have two properties:

- (A) the gravitational field of external bodies is represented in the form of tidal potentials being  $\mathcal{O}(\mathbf{X}^2)$ , where  $X^i$  are local spatial coordinates;
- (B) the internal gravitational field of the body coincides formally with the gravitational field of a corresponding isolated source provided that the tidal influence of the external matter is neglected.

The first attempt to use a version of such a local reference system to study the rotational motion of an extended body has been undertaken by Voinov [8]. Major progress to solve the problem in the Einsteinian post-Newtonian theory was achieved by Damour, Soffel and Xu [2] who used their DSX formalism aimed at constructing the local reference systems and derived the rotational equations of motion of each body of an  $N$ -body system with full multipole structure.

The aim of this paper is to derive rotational equations of motion of an arbitrarily-shaped extended body being a member of a system of  $N$  arbitrarily-shaped extended bodies in the framework of the PPN formalism with the two Eddington parameters  $\beta$  and  $\gamma$ . The PPN formalism is a phenomenological scheme giving in generic parametrized form the post-Newtonian metric tensor for an isolated material source in a wide class of metric theories of gravitation. Many aspects of testing General Relativity in the weak-field slow-motion regime are based upon the PPN formalism. On the other hand, the theory of local reference systems is also proved to be very important for physically meaningful modeling of observational data. Therefore, it is quite important to elaborate a theory of local reference systems in the framework of the PPN formalism (see also [9]).

In full detail our theory of local PPN reference systems will be discussed elsewhere. Here, we confine ourselves to a brief description of the theory and to one of its results: rotational equations of motion of a body relative to its local reference system. These equations reveal an interesting physical effect. They contain an additional torque analogous to the Nordtvedt

effect in the translational equations of motion: the torque is proportional to the Nordtvedt parameter  $\eta = 4\beta - \gamma - 3$ , to the acceleration of the body relative to the global PPN reference system and finally to some integral over the volume of the body characterizing the distribution of the internal gravitational energy inside the body.

## II. GLOBAL PPN REFERENCE SYSTEM

We begin with the global PPN metric tensor of a isolated  $N$  body system in the form (see, e.g., [10])

$$\begin{aligned} g_{00} &= -1 + \frac{2}{c^2} w(t, \mathbf{x}) - \frac{2}{c^4} \beta w^2(t, \mathbf{x}) + \mathcal{O}(c^{-5}), \\ g_{0i} &= -\frac{2(1+\gamma)}{c^3} w^i(t, \mathbf{x}) + \mathcal{O}(c^{-5}), \\ g_{ij} &= \delta_{ij} \left( 1 + \frac{2}{c^2} \gamma w(t, \mathbf{x}) \right) + \mathcal{O}(c^{-4}), \end{aligned} \quad (1)$$

where

$$w = G \int \sigma(t, \mathbf{x}') \frac{1}{|\mathbf{x} - \mathbf{x}'|} d^3x' + \frac{1}{2c^2} G \frac{\partial^2}{\partial t^2} \int \sigma(t, \mathbf{x}') |\mathbf{x} - \mathbf{x}'| d^3x' + \mathcal{O}(c^{-4}), \quad (2)$$

$$w^i = G \int \sigma^i(t, \mathbf{x}') \frac{1}{|\mathbf{x} - \mathbf{x}'|} d^3x' + \mathcal{O}(c^{-2}), \quad (3)$$

$$\sigma = \frac{1}{c^2} \left( T^{00} + \gamma T^{kk} + \frac{1}{c^2} T^{00} (3\gamma - 2\beta - 1) w \right) + \mathcal{O}(c^{-4}), \quad \sigma^i = \frac{1}{c} T^{0i} + \mathcal{O}(c^{-2}) \quad (4)$$

and  $T^{\alpha\beta}$  is the energy-momentum tensor in the global PPN reference system  $(t, x^i)$ . We retain here only two PPN parameters:  $\gamma$  and  $\beta$ . This simplest version of the PPN formalism covers, nevertheless, most viable theories of gravitation. For some technical reasons we prefer to work not in the standard PPN gauge, but in a different one, the difference affecting only the coordinate time  $t$

$$t_{\text{PPN}} = t - \frac{1}{c^4} \frac{\partial}{\partial t} \chi + \mathcal{O}(c^{-5}), \quad (5)$$

where  $t_{\text{PPN}}$  is coordinate time of the global PPN reference system in the standard PPN gauge and

$$\chi = \frac{1}{2} G \int \sigma(t, \mathbf{x}') |\mathbf{x} - \mathbf{x}'| d^3x'. \quad (6)$$

For  $\gamma = \beta = 1$  the reference system  $(t, x^i)$  is harmonic.

We have written the metric tensor of the global PPN reference system without specifying an explicit form of the energy-momentum tensor. This is no more than a formal way to specify the global PPN metric tensor. The metric (1)–(4) coincides with the version of the PPN formalism described in [10] as well as with the version discussed in [11], provided that only two parameters  $\gamma$  and  $\beta$  are retained in both versions. Effectively we consider only

those theories of gravitation which produce the metric (1)–(4) in the first post-Newtonian approximation, the energy-momentum tensor being left unspecified.

We consider the material system to consist of  $N$  bodies which represent simply spatially bounded blobs of matter. Potentials  $w$  and  $w^i$  are defined by (2)–(3) as volume integrals over the whole space. Using this fact, we split the area of integration into the volume  $V$  of the body, for which we want to construct a local reference system, and the remaining part of space. Thus, we split  $w$  and  $w^i$  into internal potentials (potentials of the body under consideration) and external ones (potentials due to the other bodies):

$$\begin{aligned} w(t, \mathbf{x}) &= w_E(t, \mathbf{x}) + \overline{w}(t, \mathbf{x}), \\ w^i(t, \mathbf{x}) &= w_E^i(t, \mathbf{x}) + \overline{w}^i(t, \mathbf{x}). \end{aligned} \quad (7)$$

### III. LOCAL PPN REFERENCE SYSTEM

From a physical point of view any reference system covering a space-time region under consideration can be used to describe physical phenomena within that region. Some reference systems, however, offer a simpler mathematical description for physical laws and make the character of physical laws more obvious. It is well known that, for example, for a massless observer one can construct a local reference system where gravitational fields appear only in tidal form. Due to the works of Brumberg and Kopeikin [3,4] and Damour, Soffel and Xu [6,7,2] we know that in the first post-Newtonian approximation of General Relativity it is possible to construct an analogue of such a local reference system for a massive extended body and that those local reference systems have two properties **A** and **B** listed in the Introduction. It is clear that the possibility to satisfy in General Relativity both properties simultaneously is closely related with the Equivalence Principle. It can be expected apriori and will be seen below that in alternative theories of gravitation where the Equivalence Principle is violated properties **A** and **B** cannot be satisfied simultaneously.

We assume the metric tensor of the local PPN reference system  $(T, X^a)$  of a selected body to be of the form

$$\begin{aligned} G_{00} &= -1 + \frac{2}{c^2} W(T, \mathbf{X}) - \frac{2}{c^4} \beta W^2(T, \mathbf{X}) + \mathcal{O}(c^{-5}), \\ G_{0a} &= -\frac{2(1+\gamma)}{c^3} W^a(T, \mathbf{X}) + \mathcal{O}(c^{-5}), \\ G_{ab} &= \delta_{ab} \left( 1 + \frac{2}{c^2} \gamma W(T, \mathbf{X}) \right) + \mathcal{O}(c^{-4}), \end{aligned} \quad (8)$$

and the local gravitational potentials  $W$  and  $W^a$  to admit a representation in the form

$$W(T, \mathbf{X}) = W_E(T, \mathbf{X}) + Q_a(T) X^a + W_T(T, \mathbf{X}) + \frac{1}{c^2} \Psi(T, \mathbf{X}), \quad (9)$$

$$W^a(T, \mathbf{X}) = W_E^a(T, \mathbf{X}) + \frac{1}{2} \varepsilon_{abc} C_b(T) X^c + W_T^a(T, \mathbf{X}). \quad (10)$$

Here, the local internal gravitational potentials  $W_E$  and  $W_E^a$  have the same functional form as their global counterparts  $w_E$  and  $w_E^i$ , but all quantities should be taken in the local reference system, i.e.

$$W_E = G \int_V \Sigma(T, \mathbf{X}') \frac{1}{|\mathbf{X} - \mathbf{X}'|} d^3 X' + \frac{1}{2c^2} G \frac{\partial^2}{\partial T^2} \int_V \Sigma(T, \mathbf{X}') |\mathbf{X} - \mathbf{X}'| d^3 X' + \mathcal{O}(c^{-4}), \quad (11)$$

$$W_E^a = G \int_V \Sigma^a(T, \mathbf{X}') \frac{1}{|\mathbf{X} - \mathbf{X}'|} d^3 X' + \mathcal{O}(c^{-2}), \quad (12)$$

$$\Sigma = \frac{1}{c^2} \left( \mathcal{T}^{00} + \gamma \mathcal{T}^{aa} + \frac{1}{c^2} \mathcal{T}^{00} (3\gamma - 2\beta - 1) W \right) + \mathcal{O}(c^{-4}), \quad \Sigma^a = \frac{1}{c} \mathcal{T}^{0a} + \mathcal{O}(c^{-2}), \quad (13)$$

where  $\mathcal{T}^{\alpha\beta}$  is the energy-momentum tensor in the local reference system. The external potentials  $W_T$  and  $W_T^a$  represent tidal field of the other bodies of the system and are assumed to be  $\sim \mathcal{O}(\mathbf{X}^2)$ . It is also assumed that  $(W_T, W_T^a)$  are functions of  $(\bar{w}, \bar{w}^i)$  and their derivatives as well as of the trajectory of the origin of the local reference system relative to the global one. Two arbitrary functions  $Q_a(T)$  and  $C_a(T)$  have a clear physical meaning which will be discussed below. Finally, the function  $\Psi(T, \mathbf{X})$  is some unknown function containing internal potentials of the central body which appear in the local PPN reference system in addition to  $W_E$  and  $W_E^a$ . Clearly the appearance of  $\Psi$  is related with a violation of the Equivalence Principle which make it impossible to satisfy simultaneously properties **A** and **B** formulated in the Introduction. By assuming that  $W_T$  and  $W_T^a$  are  $\sim \mathcal{O}(\mathbf{X}^2)$  we assume that property **A** is satisfied. Therefore, property **B** is violated which results in the appearance of  $\Psi$ .

Then, the results of the Brumberg-Kopeikin and DSX formalisms (see, e.g., Theorems 1 and 2 of [6]) allow us to write the transformations between the global and local reference systems in the form

$$T = t - \frac{1}{c^2} \left( A + v_E^i r_E^i \right) + \frac{1}{c^4} \left( B + B^i r_E^i + B^{ij} r_E^i r_E^j + C(t, \mathbf{x}) \right) + \mathcal{O}(c^{-5}), \quad (14)$$

$$X^a = R_j^a \left( r_E^j + \frac{1}{c^2} \left( \frac{1}{2} v_E^j v_E^k r_E^k + D^{jk} r_E^k + \gamma \left( r_E^j a_E^k r_E^k - \frac{1}{2} a_E^j r_E^2 \right) \right) \right) + \mathcal{O}(c^{-4}), \quad (15)$$

where  $r_E^i = x^i - x_E^i(t)$ ,  $x_E^i(t)$  is the coordinates of the origin of the local reference system relative to the global one, and  $v_E^i = dx_E^i/dt$  and  $a_E^i = d^2 x_E^i/dt^2$  are its velocity and acceleration, respectively. The functions  $A(t)$ ,  $B(t)$ ,  $B^i(t)$ ,  $B^{ij}(t) = B^{ji}(t)$ ,  $D^{ij}(t) = D^{ji}(t)$ ,  $R_j^a(t)$  (being orthogonal matrix) and  $C(t, \mathbf{x}) \sim \mathcal{O}(r_E^3)$  are some unknown functions.

The transformation rule

$$g_{\alpha\lambda}(t, \mathbf{x}) = \frac{\partial X^\mu}{\partial x^\alpha} \frac{\partial X^\nu}{\partial x^\lambda} G_{\mu\nu}(T, \mathbf{X}) \quad (16)$$

enables us then to derive or constrain the unknown functions from both the local PPN metric tensor (8)–(13) and the transformations (14)–(15)

$$\dot{A}(t) = \frac{1}{2} v_E^2 + \bar{w}(\mathbf{x}_E), \quad (17)$$

$$\dot{B}(t) = -\frac{1}{8} v_E^4 + 2(\gamma + 1) v_E^i \bar{w}^i(\mathbf{x}_E) - \left( \gamma + \frac{1}{2} \right) v_E^2 \bar{w}(\mathbf{x}_E) + \left( \beta - \frac{1}{2} \right) \bar{w}^2(\mathbf{x}_E), \quad (18)$$

$$B^i(t) = -\frac{1}{2} v_E^2 v_E^i + 2(1 + \gamma) \bar{w}^i(\mathbf{x}_E) - (2\gamma + 1) v_E^i \bar{w}(\mathbf{x}_E), \quad (19)$$

$$D^{ij} = \gamma \delta^{ij} \bar{w}(\mathbf{x}_E), \quad (20)$$

$$B^{ij}(t) = -v_E^{(i} R_{j)}^a Q_a + (1 + \gamma) \bar{w}^{(i,j)}(\mathbf{x}_E) - \gamma v_E^{(i} \bar{w}^{j)}(\mathbf{x}_E) + \frac{1}{2} \gamma \delta^{ij} \dot{\bar{w}}(\mathbf{x}_E), \quad (21)$$

$$c^2 R^a_i \dot{R}^a_j = -(1 + \gamma) \varepsilon_{ijk} R^a_k C_a - 2(1 + \gamma) \bar{w}^{[i,j]}(\mathbf{x}_E) + (1 + 2\gamma) v_E^{[i} \bar{w}_{,j]}(\mathbf{x}_E) + v_E^{[i} R^a_{j]} Q_a + \mathcal{O}(c^{-2}), \quad (22)$$

$$C_{,ii} = (\gamma - 2) \dot{a}_E^k r_E^k, \quad (23)$$

$$a_E^i(t^*) = \bar{w}_{,i}(t^*, \mathbf{x}_E(t^*)) - R^a_j Q_a(T) \left( \delta^{ij} - \frac{1}{c^2} \left( v_E^2 \delta^{ij} + (2 + \gamma) \bar{w}(\mathbf{x}_E) \delta^{ij} + \frac{1}{2} v_E^i v_E^j \right) \right) + \frac{1}{c^2} \left( 2(1 + \gamma) \dot{\bar{w}}^i(\mathbf{x}_E) + \left( \gamma v_E^2 - 2(\gamma + \beta) \bar{w}(\mathbf{x}_E) \right) \bar{w}_{,i}(\mathbf{x}_E) - (2\gamma + 1) v_E^i \dot{\bar{w}}(\mathbf{x}_E) - 2(1 + \gamma) v_E^j \bar{w}_{,i}^j(\mathbf{x}_E) - v_E^i v_E^j \bar{w}_{,j}(\mathbf{x}_E) \right) + \mathcal{O}(c^{-4}), \quad (24)$$

$$T = t^* - \frac{1}{c^2} A(t^*) + \mathcal{O}(c^{-4}). \quad (25)$$

For any function  $A(\mathbf{x}_E)$  means  $A(t, \mathbf{x}_E(t))$ . Parentheses and brackets around a group indices signify, respectively, symmetric and antisymmetric parts of the corresponding expressions:  $A^{(ij)} = \frac{1}{2} (A^{ij} + A^{ji})$  and  $A^{[ij]} = \frac{1}{2} (A^{ij} - A^{ji})$ , etc. The moment of time  $t^*$  which appears in (24) is defined by (25). The function  $C(t, \mathbf{x})$  is not fixed completely, but should only satisfy (23). It is clear, however, that the PPN equations of motion do not depend on  $C(t, \mathbf{x})$ . Equations (24)–(25) represent equations of motion of the origin of the local reference system relative to the global one. For  $Q_a = 0$  the equations coincide with the geodesic equations in the external potentials  $\bar{w}$  and  $\bar{w}^i$ .

The external potentials  $W_T$  and  $W_T^a$  represent the tidal fields of external masses and are given by

$$W_T(T, \mathbf{X}) = \bar{w}(t, \mathbf{x}) - \bar{w}(\mathbf{x}_E) - \bar{w}_{,j}(\mathbf{x}_E) r_E^j + \frac{1}{c^2} \left( -2(1 + \gamma) v_E^i \left( \bar{w}^i(t, \mathbf{x}) - \bar{w}^i(\mathbf{x}_E) - \bar{w}_{,j}^i(\mathbf{x}_E) r_E^j \right) + (1 + \gamma) v_E^2 W_T + (1 + \gamma) \dot{\bar{w}}^{i,j}(\mathbf{x}_E) r_E^i r_E^j + \frac{1}{2} \gamma \ddot{\bar{w}}(\mathbf{x}_E) r_E^2 + \left( \frac{1}{2} - \beta - \gamma \right) (a_E^i r_E^i)^2 + (1 - 2\beta - 2\gamma) Q_a X^a a_E^i r_E^i - \gamma v_E^i r_E^i \bar{w}_{,j}(\mathbf{x}_E) r_E^j + \frac{1}{2} \gamma r_E^2 R^a_i a_E^i Q_a + \frac{\partial}{\partial T} C(T, \mathbf{X}) + 2(1 - \beta) \left( \bar{w}(\mathbf{x}_E) + a_E^i r_E^i \right) W_T \right) + \mathcal{O}(c^{-4}), \quad (26)$$

$$W_T^a(T, \mathbf{X}) = R^a_i \left\{ \bar{w}^i(t, \mathbf{x}) - \bar{w}^i(\mathbf{x}_E) - \bar{w}_{,j}^i(\mathbf{x}_E) r_E^j - v_E^i W_T(T, \mathbf{X}) + \frac{1}{2(1 + \gamma)} \left( \gamma \left( r_E^i a_E^j r_E^j - \frac{1}{2} a_E^i r_E^2 \right) - C_{,i}(T, \mathbf{X}) \right) \right\} + \mathcal{O}(c^{-2}). \quad (27)$$

Finally, the function  $\Psi$  reads

$$\Psi(T, \mathbf{X}) = -\eta \left( w_E(t, \mathbf{x}) \left( \bar{w}(\mathbf{x}_E) + a_E^i r_E^i \right) - \chi^E_{,i}(t, \mathbf{x}) a_E^i \right) + \mathcal{O}(c^{-2}), \quad (28)$$

$\eta = 4\beta - \gamma - 3$  being the Nordtvedt parameter, and  $\chi^E(t, \mathbf{x})$  is defined by the integral (6) taken over the volume  $V$  of the central body. The presence of this function reflects the violation of property **B** in the local reference system  $(T, X^a)$ . Property **A** is satisfied, since both  $W_T$  and  $W_T^a$  are  $\sim \mathcal{O}(\mathbf{X}^2)$ . One can show that by changing the transformations

(14)–(25) one can construct another version of the local reference system where property **B** is valid, but property **A** is violated. One can also show that it is impossible to satisfy both properties simultaneously. This fact is a direct consequence of the violation of the Equivalence Principle for  $\eta \neq 0$ .

The function  $Q_a(T)$  characterizes the world line of the origin of the local reference system.  $Q_a(T)$  represents the acceleration of the instantaneous locally inertial reference system (whose origin coincides with that of the local reference system at a given moment of time) expressed in the local reference system. E.g., for  $Q_a = 0$  the origin moves along a geodesic in the external gravitational field. The value  $Q_a$  can be also chosen in such a way that the multipole expansion of the internal gravitational field  $W_E$  does not have a dipole component. The function  $C_a(T)$  describes the spatial orientation of the local reference system with respect to the global one. Its relation to the orthogonal matrix  $R^a_i$  is defined by (22). One can choose  $C_a(T)$  so that  $R^a_i = \delta^a_i$  and the resulting local reference system does not rotate relative to the global one. Another possible choice is  $C_a = 0$  resulting in a dynamically nonrotating local reference system and the orthogonal matrix  $R^a_i$  in the transformations of the spatial coordinates represents the well-known de Sitter, Lense-Thirring and Thomas precessions.

#### IV. ROTATIONAL EQUATIONS OF MOTIONS

The local reference system of an extended massive body described above allows us to derive the rotational equations of motion of the body in the framework of the PPN formalism. From the equation (here,  $\mathcal{G}$  is the determinant of the local metric tensor)

$$\varepsilon_{abc} \int_V (-\mathcal{G}) X^b \mathcal{T}^{c\beta}_{;\beta} d^3X = 0, \quad (29)$$

which is valid due to the local equations of motion

$$\mathcal{T}^{\alpha\beta}_{;\beta} = 0, \quad (30)$$

in analogy to General Relativity (see [2,12]) one can derive the rotational equations of motion for the body

$$\frac{d}{dT} S^a = L^a_{\text{inertial}} + L^a_{\text{T}} + L^a_{\text{Nor}} + \mathcal{O}(c^{-4}), \quad (31)$$

where the PPN spin  $S^a$  is defined by

$$S^a = \varepsilon_{abc} \int_V X^b p^c(T, \mathbf{X}) d^3X + \mathcal{O}(c^{-4}), \quad (32)$$

$$p^a = \Sigma^a \left(1 + \frac{5\gamma - 1}{c^2} W\right) - \frac{1}{2c^2} G \Sigma \int_V \Sigma^b(T, \mathbf{X}') \frac{(4\gamma + 3) \delta^{ab} + n^a n^b}{|\mathbf{X} - \mathbf{X}'|} d^3X' + \mathcal{O}(c^{-4}), \quad (33)$$

$$n^a = \frac{X^a - X'^a}{|\mathbf{X} - \mathbf{X}'|}, \quad (34)$$

the PPN inertial and tidal torques read

$$L_{\text{inertial}}^a = \varepsilon_{abc} \int_V X^b \left( \Sigma Q_c + \frac{1}{c^2} 2(1 + \gamma) \varepsilon_{cde} \left( C_d \Sigma^e + \frac{1}{2} \Sigma \dot{C}_d X^e \right) \right) d^3 X, \quad (35)$$

$$L_{\text{T}}^a = \varepsilon_{abc} \int_V X^b \left( \Sigma W_{\text{T},c} + \frac{1}{c^2} 2(1 + \gamma) \left( \Sigma \frac{\partial}{\partial T} W_{\text{T}}^c + \Sigma^d (W_{\text{T},d}^c - W_{\text{T},c}^d) \right) \right) d^3 X + \mathcal{O}(c^{-4}), \quad (36)$$

and the additional torque  $L_{\text{Nor}}^a$  is defined as

$$L_{\text{Nor}}^a = \frac{1}{c^2} \eta \varepsilon_{abc} \int_V \Sigma X^b \Psi_{,c} d^3 X = \frac{1}{c^2} \eta \varepsilon_{abc} \Omega_{\text{E}}^b a_{\text{E}}^c + \mathcal{O}(c^{-4}), \quad (37)$$

$$\Omega_{\text{E}}^a = -\frac{1}{2} \int_V \Sigma W_{\text{E}} X^a d^3 X + \mathcal{O}(c^{-2}). \quad (38)$$

The inertial torque is due to inertial forces dependent on the choice of  $Q_a$  and  $C_a$ . The PPN tidal torque  $L_{\text{T}}^a$  is due to external gravitational potentials  $W_{\text{T}}$  and  $W_{\text{T}}^a$ . It vanishes for an isolated body and is proportional to the square of the spatial size of the body. The additional torque  $L_{\text{Nor}}^a$  represents an analogy to the Nordtvedt effect (known from the PPN translational equations of motion of extended self-gravitating bodies) in the rotational equations of motion.  $L_{\text{Nor}}^a$  is proportional to the Nordtvedt parameter  $\eta$ , which is not zero in a particular theory of gravitation only if that theory leads to the violation of the Equivalence Principle. The effect results from the difference between the center of inertial mass and that of gravitational mass (the latter involves also the gravitational binding energy). The Nordtvedt effect consists actually in the fact that the two forms of mass-energy experience different accelerations provided that the Strong Equivalence Principle is violated. This fact results in the additional torque  $L_{\text{Nor}}^a$ . This additional torque in the rotational equations of motion enables us in principle to test the Strong Equivalence Principle with observations of rotational motion of celestial bodies: if  $4\beta - \gamma - 3 \neq 0$  rotational motion of a body depends on its acceleration relative to the global reference system.  $L_{\text{Nor}}^a$  is proportional also to the integral  $\Omega_{\text{E}}^a$  which is an analogy to the internal gravitational energy of the body  $\Omega_{\text{E}}$  appearing in the PPN translational equations of motion.  $\Omega_{\text{E}}^a$  obviously vanishes for spherically symmetric bodies. Roughly speaking, for  $\Omega_{\text{E}}^a$  to be non-zero the body should be non-symmetric with respect to its center of mass (for example,  $\Omega_{\text{E}}^a \neq 0$  for a homogeneous semi-sphere). This implies that  $\Omega_{\text{E}}^a$  is quite small for typical celestial bodies. For that reason it is unclear if this “new effect” appearing in the metric theories of gravity will lead to measurable consequences.

More details about the local PPN reference system as well as about various equations of motion and their multipole expansions will be published elsewhere.

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